INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE

M. Stat.-I Year

Elective - Discrete Mathematics

Time: 2 **Hours** | Mid-Semester Examination | **Marks : 30**

Answer all the questions

1. Suppose G is a simple graph with at least 10 edges, 5 vertices of degree 3 and the remaining vertices have degree less than 3. Find the minimum number of vertices G can have. Draw such a graph. (2 Marks)

2. Show that a graph G is connected iff every entry in $I + A + A^2 + \cdots + A^{n-1}$ is non zero, where A denotes the adjacency matrix of G. (3 Marks)

3. If G is a simple graph with $\delta(G) \geq 3$, show that it contains a cycle of even length. (2 Marks)

4. A highway department team is entrusted with the job of inspecting for the fallen trees on all the roads and report back. The road map is as shown in the graph G_1 given below, where an integer against an edge denotes the road length. Find the shortest route and the shortest distance that the team has to travel. (5 Marks)

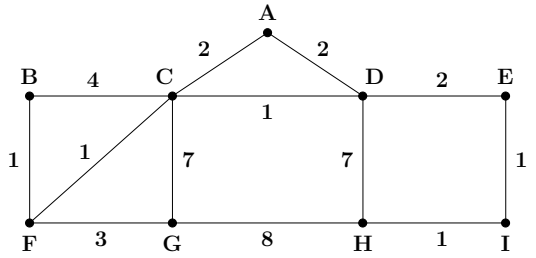

Figure : G_1

5. Using Prim's algorithm, find a minimum spanning tree of the graph G_2 shown below. Draw all the intermediate trees generated by the algorithm. (3 Marks)

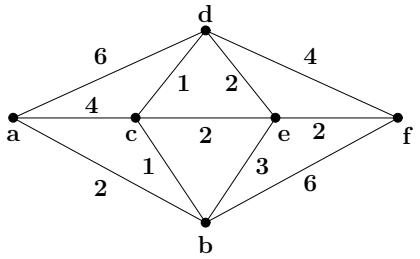

Figure : G_2

6. Draw the tree associated with the Prüfer code (3, 6, 2, 5, 6, 3, 6, 3). (2 Marks)

7. Find a necessary and sufficient condition for the graph $K_{m,n}$ (in terms of m and n) to be (i) Eulerian, and (ii) Hamiltonian. Justify your answer. (2 Marks)

8. If G is a graph with $n \geq 3$ and $m \geq \dfrac{n^2 - 3n + 6}{2}$, then show that G is Hamiltonian. Also, show by an example that the hypothesis of this statement can not be improved. (3 Marks)

9. State and prove the five color theorem on planar graphs. (4 Marks)

10. Find the value of the Ramsey number $r(3, 4)$. Justify your answer. (4 Marks)

— ALL THE BEST —